Gaussian Belief Space Planning for Imprecise Articulated Robots

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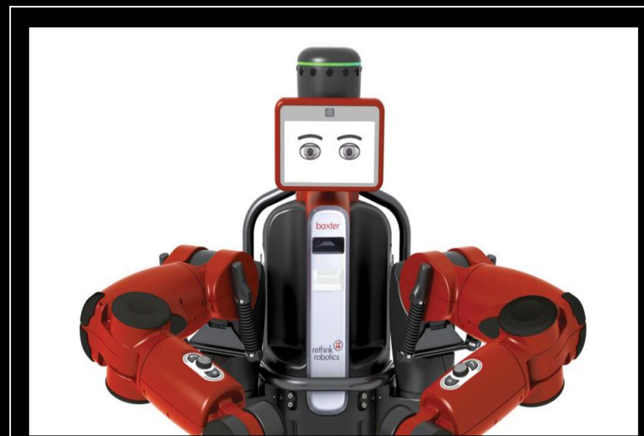
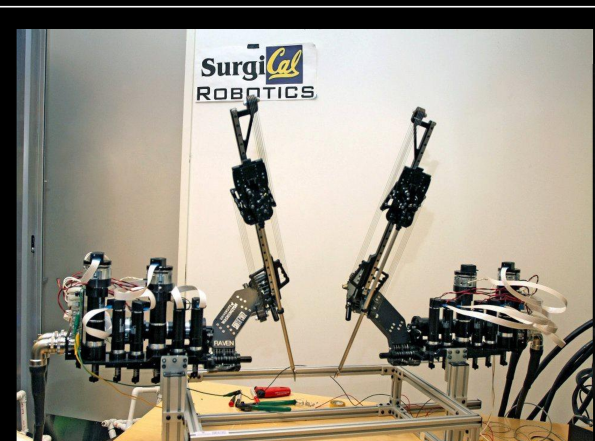
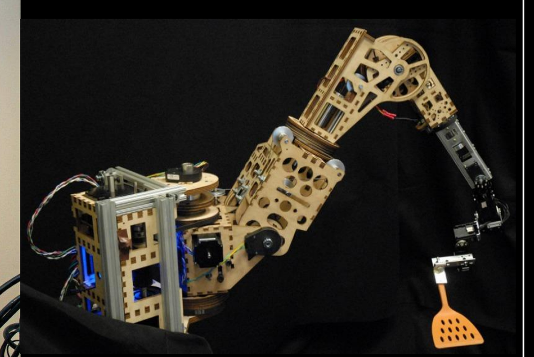


Baxter robot [Rethink Robotics]



Raven surgical robot [Rosen et al.]



Low-cost arm [Quigley et al.]

Motivation: Facilitate reliable operation of cost-effective robots that use:

- Imprecise actuation mechanisms such as serial elastic actuators and cables
- Inaccurate encoders and sensors such as gyros and accelerometers to sense robot state

Problem: Reliable manipulation and navigation requires such robots to explicitly perform information gathering actions to minimize effects of uncertainty

Formally, this can be modeled as a Partially Observable Markov Decision Process (POMDP); computing globally optimal solutions (policies) is computationally intractable

Approach: Compute locally optimal trajectories in *belief* space (space of probability distributions over states) [2]

Contributions:

- Prior work approximates robot geometry as points or spheres; we consider articulated robots
- *Sigma Hulls* for probabilistic collision avoidance
- Model predictive control (MPC) during execution in Gaussian belief space using efficient SQP-based trajectory optimization methods [3]

Trajectory optimization in Gaussian belief space:

Gaussian belief state in joint space: (mean and square root of covariance) $\mathbf{b}_t = \begin{bmatrix} \boldsymbol{\mu}_t \\ \sqrt{\Sigma_t} \end{bmatrix}$

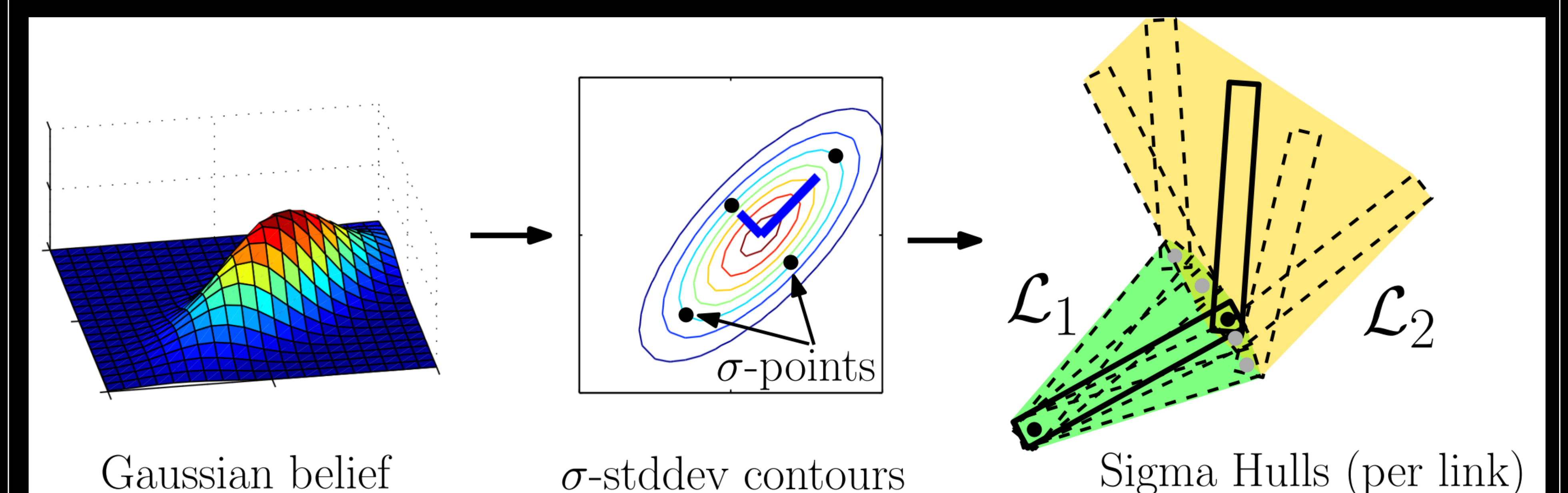
Optimization problem:

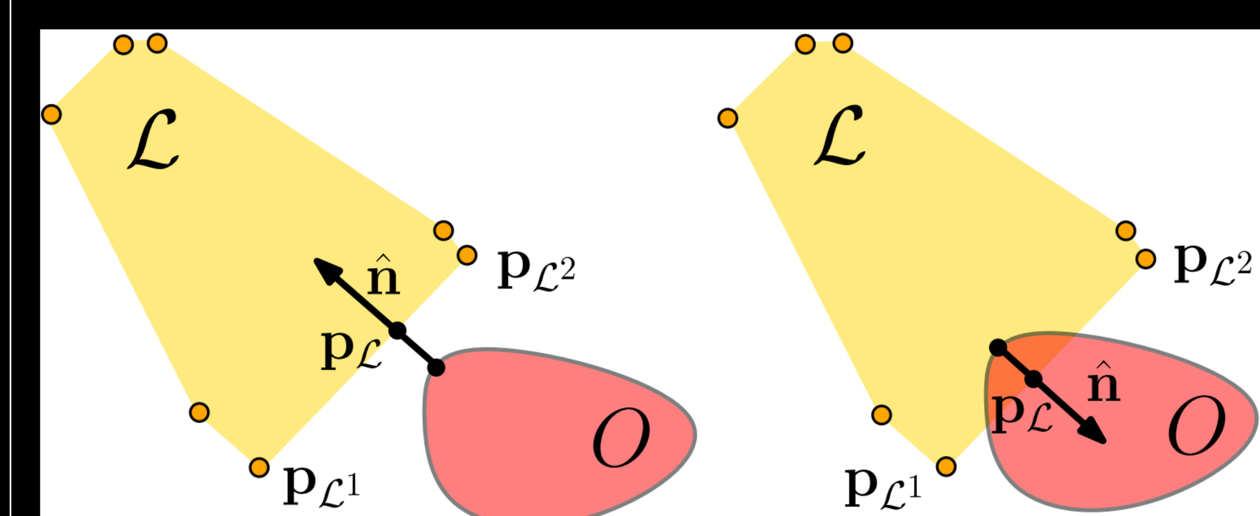
Variables (beliefs, control inputs): $\mathbf{b}_1, \ldots, \mathbf{b}_T, \mathbf{u}_1, \ldots, \mathbf{u}_{T-1}$

Minimize: $\| \boldsymbol{\mu}_T - \boldsymbol{\mu}_{\text{target}} \| + \sum_{t=1}^{T} \text{tr}[\Sigma_t]$

Subject to constraints:

- Belief dynamics (Unscented Kalman Filter [1]) are satisfied: $\mathbf{b}_{t+1} = \text{UKF}(\mathbf{b}_t, \mathbf{u}_t)$
- Control inputs are feasible: $\mathbf{u}_t \in F_{\mathcal{U}}$
- Trajectory is σ-standard deviations safe (all pairs of links and obstacles): $\texttt{sign\_dist}(\mathcal{L}_i, O_j, \sigma) \geq 0 \;\; \forall t \in \{1, \ldots, T\}$





$\texttt{sign\_dist}_{\mathcal{L}O}(\mathbf{b}_t) > 0$ $\texttt{sign\_dist}_{\mathcal{L}O}(\mathbf{b}_t) < 0$

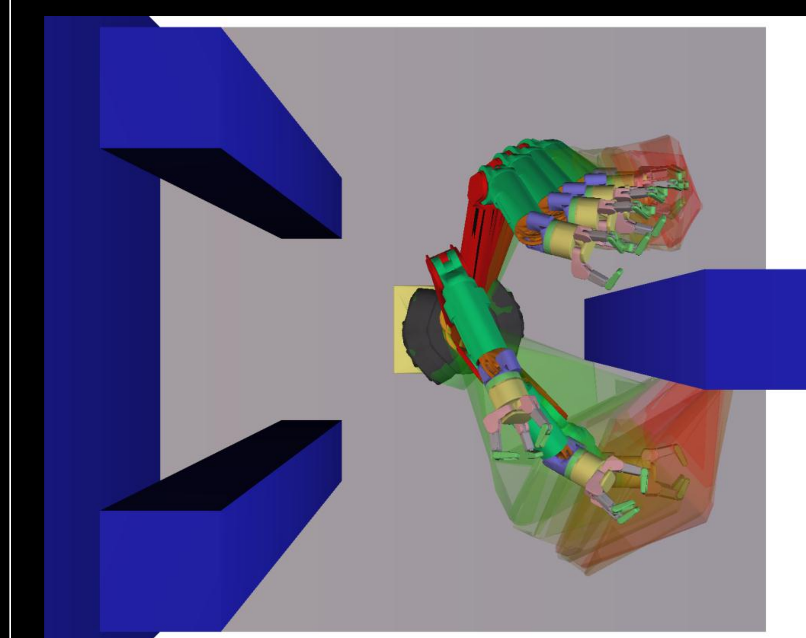
Sigma Hulls never explicitly constructed!

Efficiently compute $\texttt{sign\_dist}$ using support mappings (GJK, EPA)

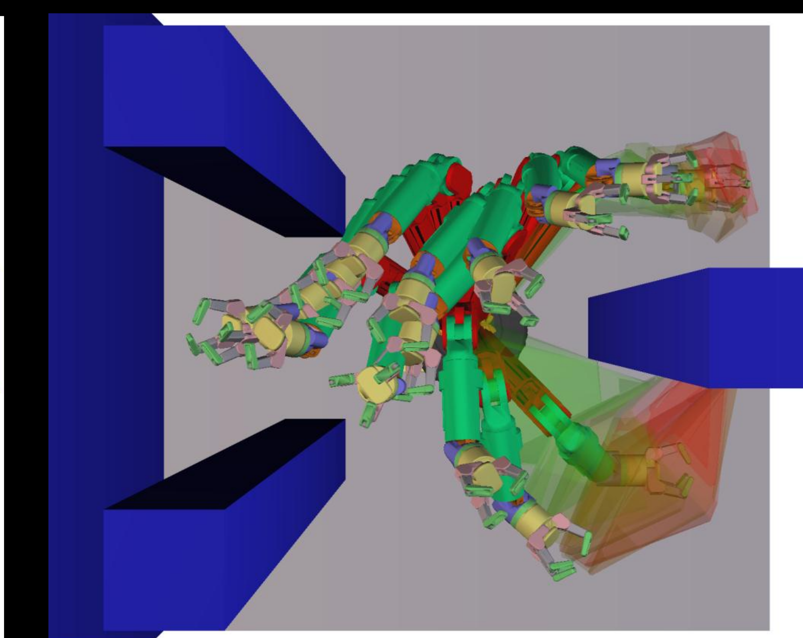
Linearized constraint (for optimization): $\texttt{sign\_dist}_{\mathcal{L}O}(\hat{\mathbf{b}}_t) + S_t(\mathbf{b}_t - \hat{\mathbf{b}}_t) \geq 0$

$$S_t = \frac{\partial \texttt{sign\_dist}_{\mathcal{L}O}}{\partial \mathbf{b}}(\hat{\mathbf{b}}_t) \approx -\hat{\mathbf{n}}^T \frac{\partial \mathbf{p}_{\mathcal{L}}}{\partial \mathbf{b}}(\hat{\mathbf{b}}_t) \equiv -\hat{\mathbf{n}}^T \left(\alpha \frac{\partial \mathbf{p}_{L^1}}{\partial \mathbf{b}}(\hat{\mathbf{b}}_t) + (1-\alpha) \frac{\partial \mathbf{p}_{L^2}}{\partial \mathbf{b}}(\hat{\mathbf{b}}_t)\right)$$

Scenario: Imprecise 7-DOF robot operating in a constrained environment. The robot localizes itself based on distance measurements from a wall using a sensor mounted on the end-effector

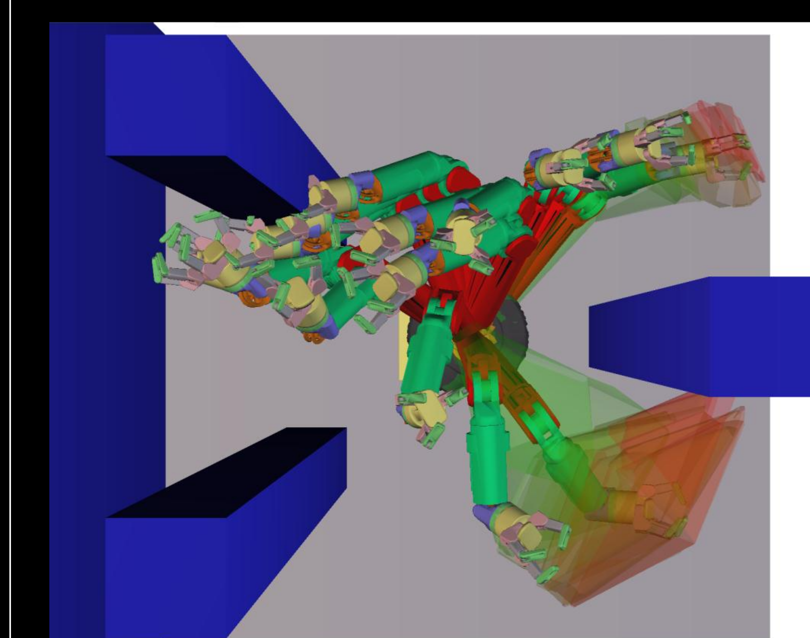


Uncertainty unaware RRT plan

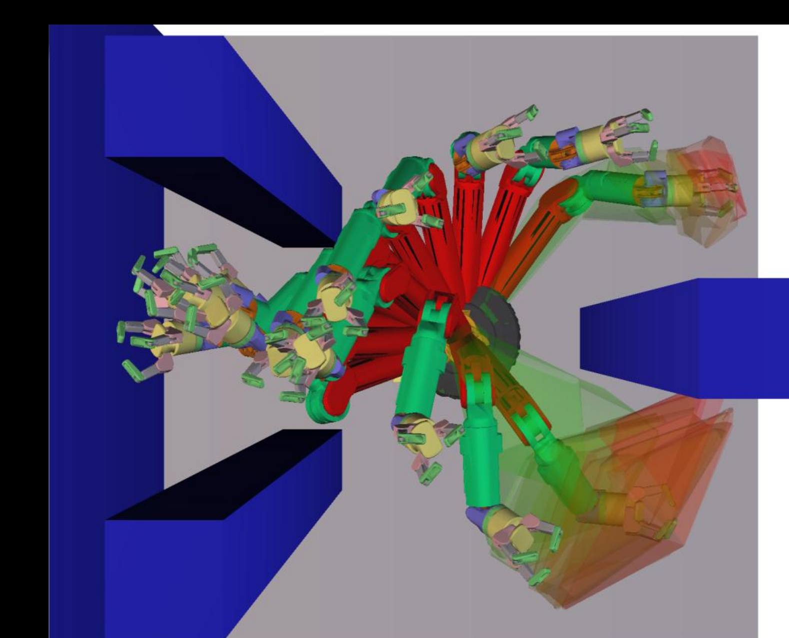


Belief space plan (35 dimensional space, ~3s)

Re-plan at every time step during execution (MPC)



Open-loop execution (probability of collision: 83%)



Closed-loop execution (probability of collision: 6%)

Future Work:

- Planning in uncertain environments
- Non-Gaussian belief spaces
- Physical experiments with the Raven surgical robot

[1] S. J. Julier and J. K. Uhlmann, "Unscented filtering and nonlinear estimation," Proc. of the IEEE, vol. 92, no. 3, pp. 401–422, 2004.
[2] R. Platt, R. Tedrake, L. Kaelbling, and T. Lozano-Perez, "Belief Space Planning assuming Maximum Likelihood Observations," in Robotics: Science and Systems (RSS), 2010. Applications and Visions, 2011, ch. 8, pp. 159–197.
[3] J. Schulman, A. Lee, H. Bradlow, I. Awwal, and P. Abbeel, "Finding Locally Optimal, Collision-Free Trajectories with Sequential Convex Optimization," in Robotics: Science and Systems (RSS), To appear, 2013.