Model-Based Reinforcement Learning

CS 294-112: Deep Reinforcement Learning

Sergey Levine

Class Notes

1. Homework 3 due in one week
   - Don’t put it off! It takes a while to train.
2. Project proposal due in two weeks!

Overview

1. Last lecture: choose good actions autonomously by backpropagating (or planning) through *known* system dynamics (e.g. known physics)
2. Today: what do we do if the dynamics are *unknown*?
   a. Fitting global dynamics models ("model-based RL")
   b. Fitting local dynamics models
3. Wednesday: combining optimal control and policy search to train neural network policies with the aid of optimal control

Today's Lecture

1. Overview of model-based RL
   - Learn only the model
   - Learn model & policy
2. What kind of models can we use?
3. Global models and local models
4. Learning with local models and trust regions

- Goals:
  - Understand the terminology and formalism of model-based RL
  - Understand the options for models we can use in model-based RL
  - Understand practical considerations of model learning

Why learn the model?

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \text{ s.t. } \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$$

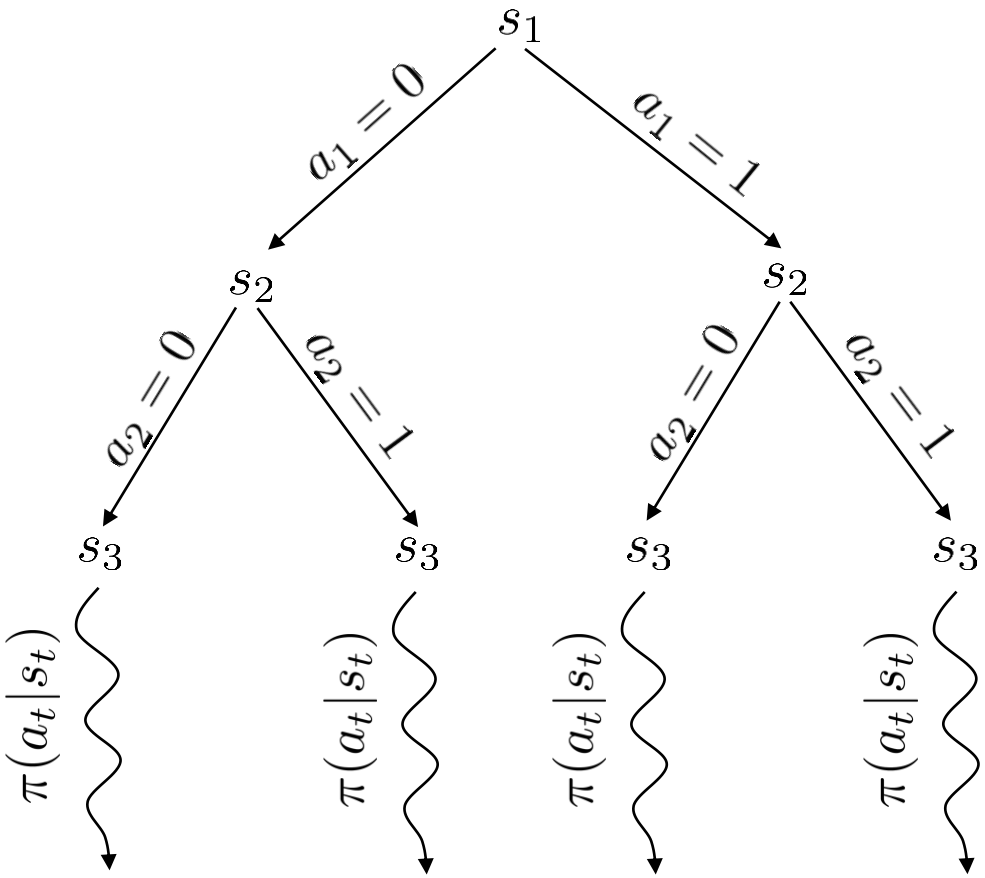
$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} c(\mathbf{x}_1, \mathbf{u}_1) + c(f(\mathbf{x}_1, \mathbf{u}_1), \mathbf{u}_2) + \cdots + c(f(f(\ldots)\ldots), \mathbf{u}_T)$$

usual story: differentiate via backpropagation and optimize!

need $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}, \frac{dc}{d\mathbf{x}_t}, \frac{dc}{d\mathbf{u}_t}$

Why learn the model?

s_t a_t



Why learn the model?

If we knew $f(\mathbf{s}_t, \mathbf{a}_t) = \mathbf{s}_{t+1}$, we could use the tools from last week.

(or $p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$ in the stochastic case)

So let's learn $f(\mathbf{s}_t, \mathbf{a}_t)$ from data, and *then* plan through it!

model-based reinforcement learning version 0.5:

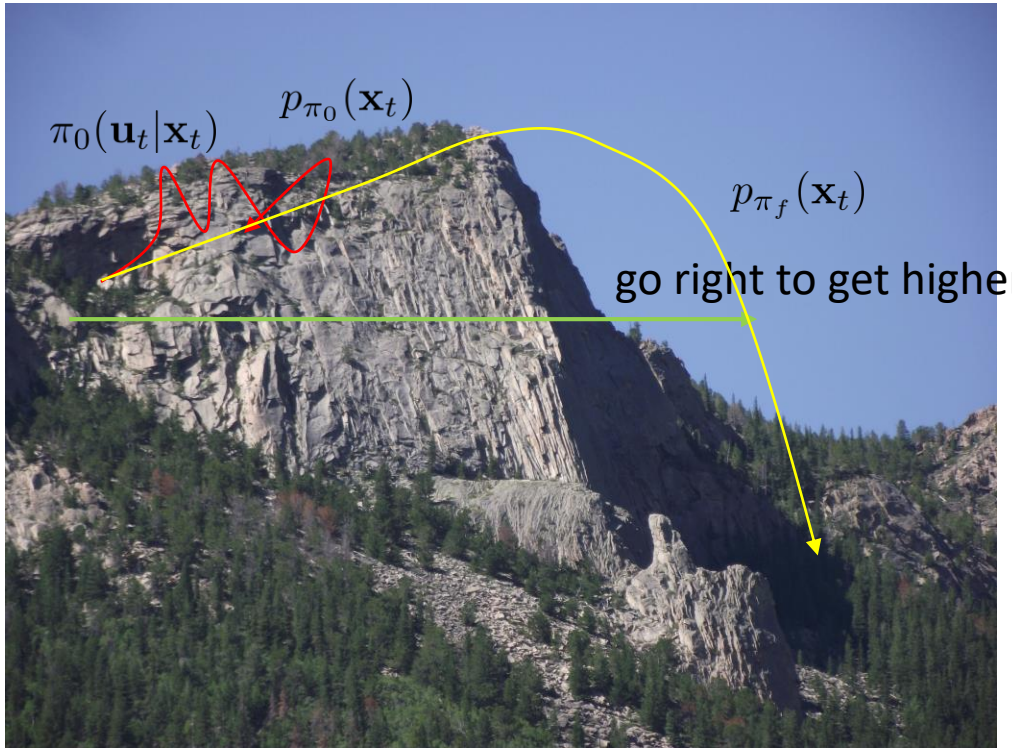
1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions

Does it work? Yes!

- Essentially how system identification works in classical robotics
- Some care should be taken to design a good base policy
- Particularly effective if we can hand-engineer a dynamics representation using our knowledge of physics, and fit just a few parameters

Does it work?

No!



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2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions

$$p_{\pi_f}(\mathbf{s}_t) \neq p_{\pi_0}(\mathbf{s}_t)$$

- Distribution mismatch problem becomes exacerbated as we use more expressive model classes

Can we do better?

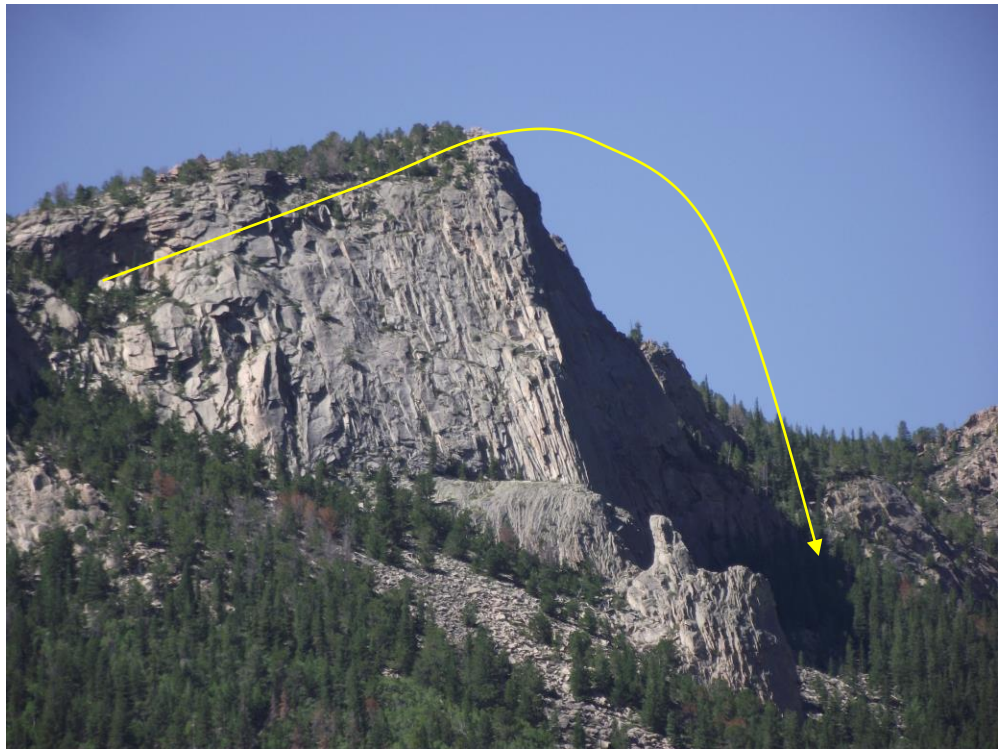
can we make $p_{\pi_0}(\mathbf{s}_t) = p_{\pi_f}(\mathbf{s}_t)$?

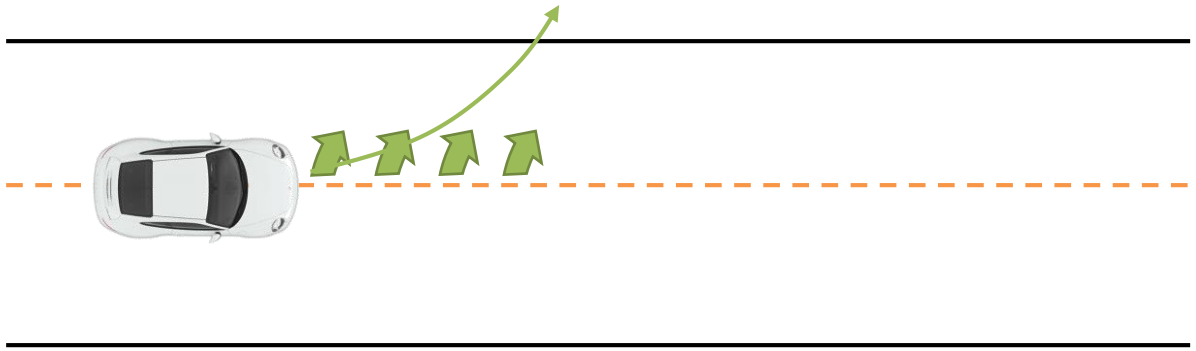
where have we seen that before? need to collect data from $p_{\pi_f}(\mathbf{s}_t)$

model-based reinforcement learning version 1.0:

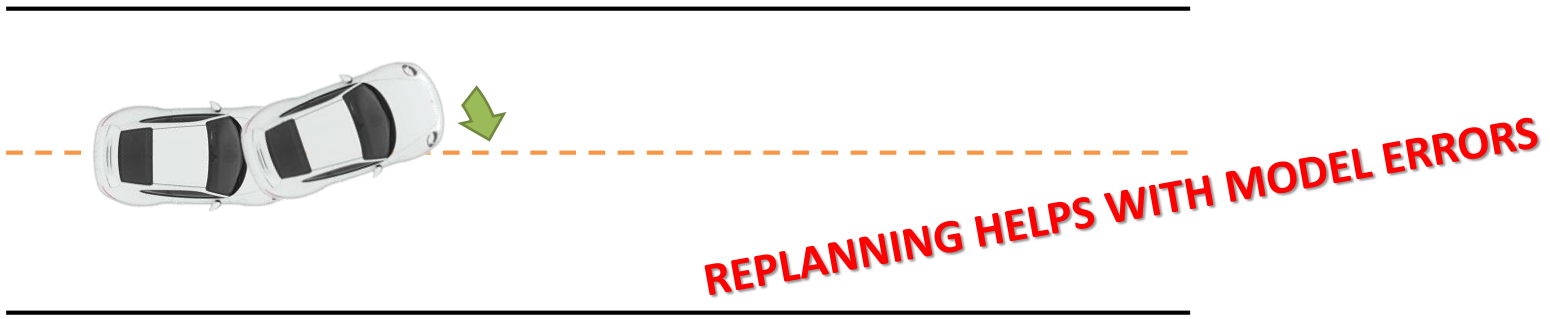
1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute those actions and add the resulting data $\{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_j\}$ to $\mathcal{D}$

What if we make a mistake?





Can we do better?



model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
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3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

every N steps

This will be on HW4!

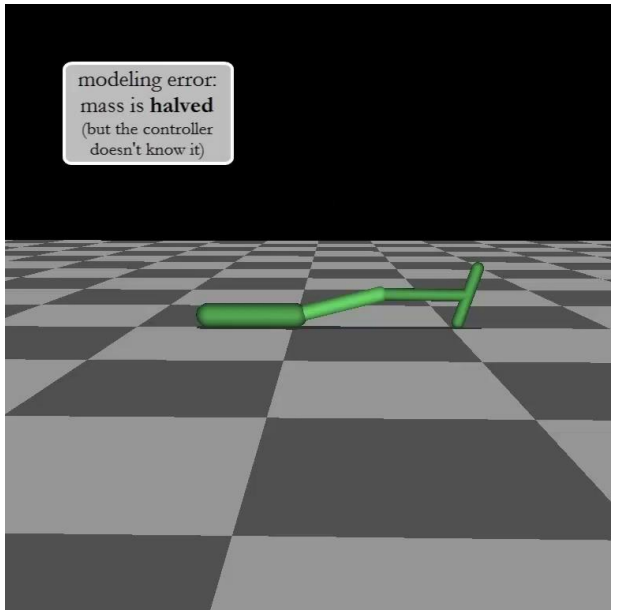
How to replan?

model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

every N steps

- The more you replan, the less perfect each individual plan needs to be
- Can use shorter horizons
- Even random sampling can often work well here!

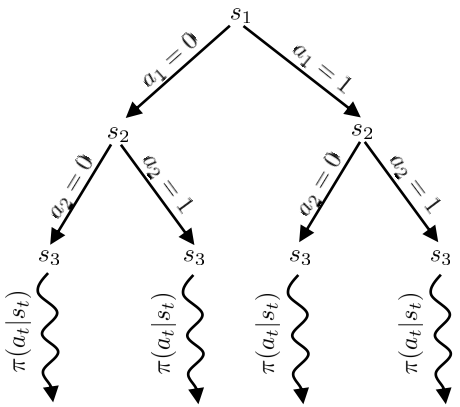


That seems like a lot of work…

model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. backpropagate through $f(\mathbf{s}, \mathbf{a})$ to choose actions (e.g. using iLQR)
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

every N steps



Deep Learning for Real-Time Atari Game Play Using Offline Monte-Carlo Tree Search Planning

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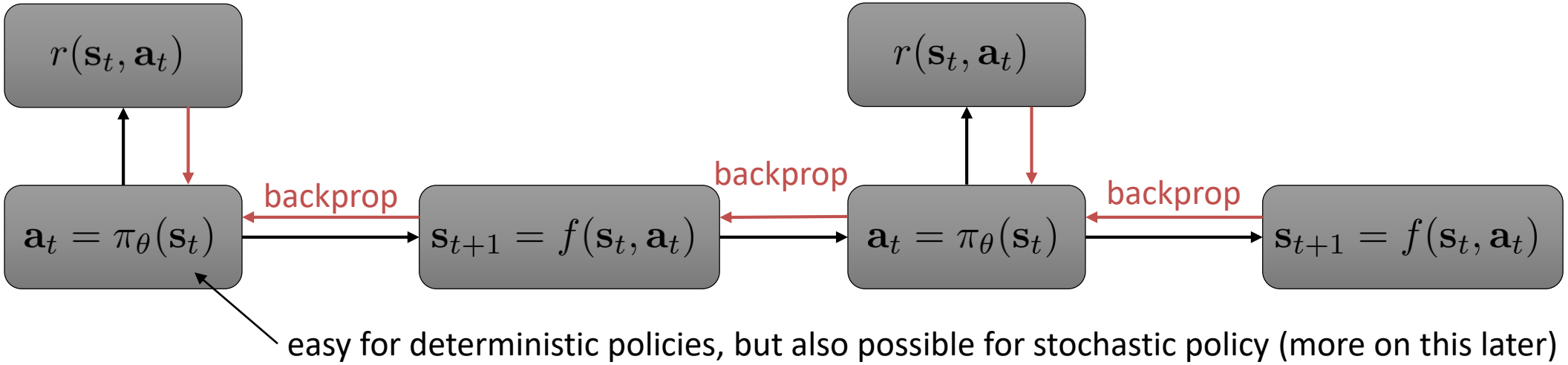
Backpropagate directly into the policy?



model-based reinforcement learning version 2.0:

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2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. backpropagate through $f(\mathbf{s}, \mathbf{a})$ into the policy to optimize $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
4. run $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$, appending the visited tuples $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to $\mathcal{D}$

Summary

- Version 0.5: collect random samples, train dynamics, plan
  - Pro: simple, no iterative procedure
  - Con: distribution mismatch problem
- Version 1.0: iteratively collect data, replan, collect data
  - Pro: simple, solves distribution mismatch
  - Con: open loop plan might perform poorly, esp. in stochastic domains
- Version 1.5: iteratively collect data using MPC (replan at each step)
  - Pro: robust to small model errors
  - Con: computationally expensive, but have a planning algorithm available
- Version 2.0: backpropagate directly into policy
  - Pro: computationally cheap at runtime
  - Con: can be numerically unstable, especially in stochastic domains (more on this later)

Case study: model-based policy search with GPs

Learning to Control a Low-Cost Manipulator using Data-Efficient Reinforcement Learning

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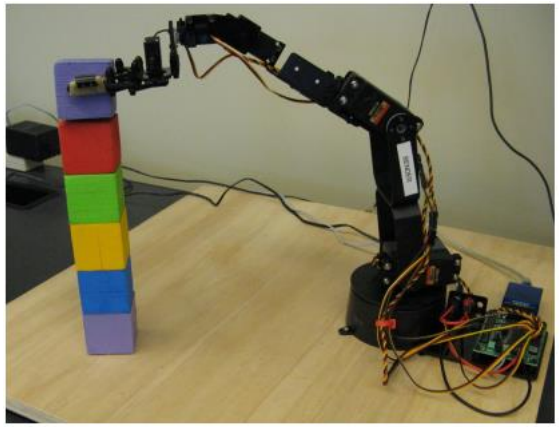

Case study: model-based policy search with GPs

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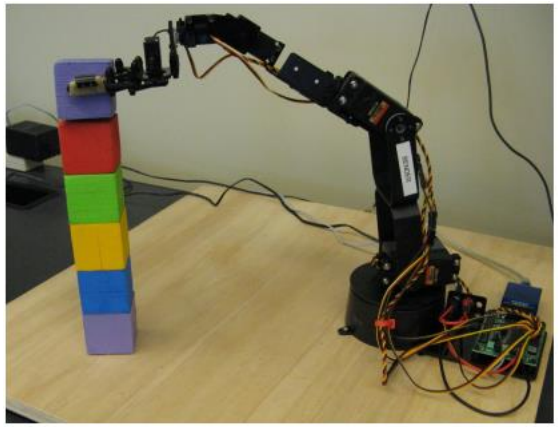


1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn GP dynamics model $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ to maximize $\sum_i \log p(\mathbf{s}'_i|\mathbf{s}_i, \mathbf{a}_i)$
3. backpropagate through $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ into the policy to optimize $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
4. run $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$, appending the visited tuples $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to $\mathcal{D}$

Case study: model-based policy search with GPs

3. backpropagate through $p(\mathbf{s}'|\mathbf{s},\mathbf{a})$ into the policy to optimize $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$

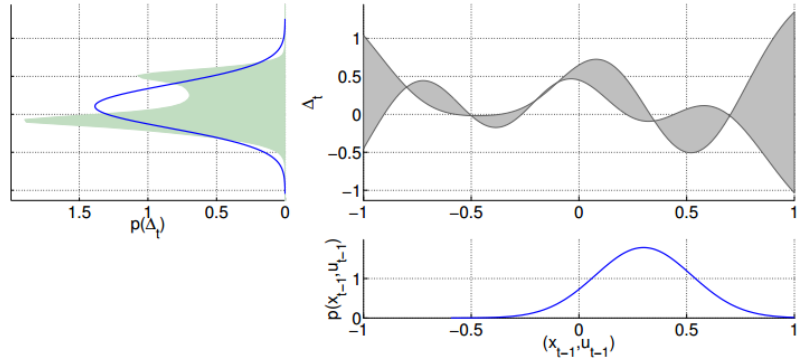
Given $p(\mathbf{s}_t)$, use $p(\mathbf{s}'|\mathbf{s},\mathbf{a})$ to compute $p(\mathbf{s}_{t+1})$

If $p(\mathbf{s}_t)$ is Gaussian, we can get a (non-Gaussian) $\bar{p}(\mathbf{s}_{t+1})$ in closed form

Project non-Gaussian $\bar{p}(\mathbf{s}_{t+1})$ to Gaussian $p(\mathbf{s}_{t+1})$ using moment matching

$E_{\mathbf{s}\sim p(\mathbf{s})}[c(\mathbf{s})]$ easy if c is nice and $p(\mathbf{s})$ Gaussian

Write $\sum_t E_{\mathbf{s}\sim p(\mathbf{s}_t)}[r(\mathbf{s}_t)]$ and differentiate

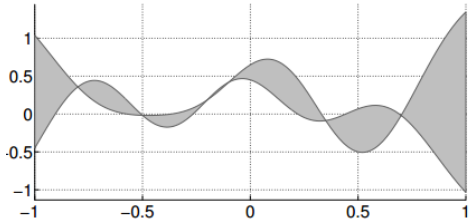


Marc Peter Deisenroth, Carl Edward Rasmussen, Dieter Fox

Learning to Control a Low-Cost Manipulator using Data-efficient Reinforcement Learning

What kind of models can we use?

Gaussian process



GP with input $(\mathbf{s}, \mathbf{a})$ and output $\mathbf{s}'$

Pro: very data-efficient

Con: not great with non-smooth dynamics

Con: very slow when dataset is big

neural network

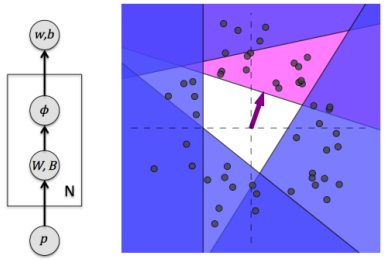


image: Punjani & Abbeel '14

Input is $(\mathbf{s}, \mathbf{a})$, output is $\mathbf{s}'$

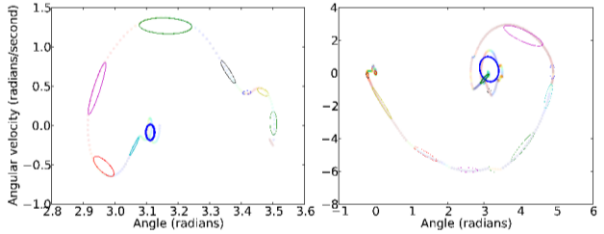
Euclidean training loss corresponds to Gaussian $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$

More complex losses, e.g. output parameters of Gaussian mixture

Pro: very expressive, can use lots of data

Con: not so great in low data regimes

other



GMM over $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ tuples

Train on $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$, condition to get $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$

For i^{th} mixture element, $p_i(\mathbf{s}, \mathbf{a})$ gives region where the mode $p_i(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ holds

other classes: domain-specific models (e.g. physics parameters)

video prediction?
more on this later
in the course

Neural Network Dynamics for Model-Based Deep Reinforcement Learning with Model-Free Fine-Tuning

Anusha Nagabandi, Gregory Kahn, Ronald S. Fearing, Sergey Levine
University of California, Berkeley

model-based reinforcement learning version 1.5:

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2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions (random sampling)
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

Model-Based Burn-In for Sample-Efficient Deep Reinforcement Learning

Break

The trouble with global models

Global model: $f(\mathbf{s}_t, \mathbf{a}_t)$ represented by a big neural network

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute those actions and add the resulting data $\{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_j\}$ to $\mathcal{D}$

- Planner will seek out regions where the model is erroneously optimistic
- Need to find a very good model in most of the state space to converge on a good solution

The trouble with global models

- Planner will seek out regions where the model is erroneously optimistic
- Need to find a very good model in most of the state space to converge on a good solution
- In some tasks, the model is much more complex than the policy

Local models

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \text{ s.t. } \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$$

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} c(\mathbf{x}_1, \mathbf{u}_1) + c(f(\mathbf{x}_1, \mathbf{u}_1), \mathbf{u}_2) + \cdots + c(f(f(\ldots)\ldots), \mathbf{u}_T)$$

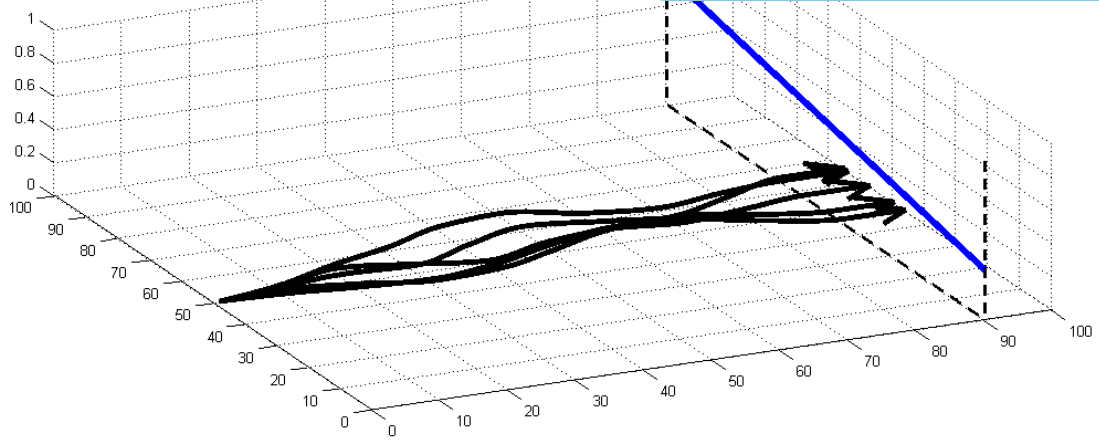
usual story: differentiate via backpropagation and optimize!

need $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}, \frac{dc}{d\mathbf{x}_t}, \frac{dc}{d\mathbf{u}_t}$

Local models

need $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}, \frac{dc}{d\mathbf{x}_t}, \frac{dc}{d\mathbf{u}_t}$

idea: just fit $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}$ around current trajectory or policy!



LQR gives us a linear feedback controller

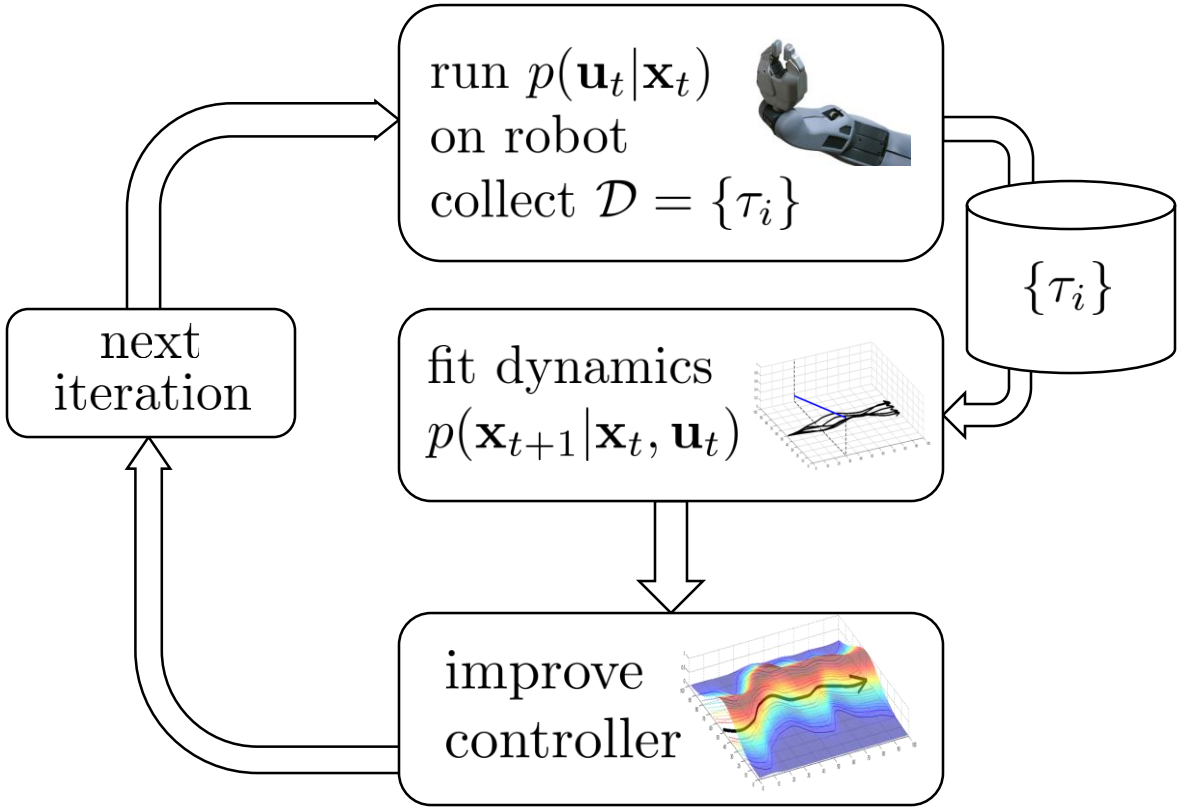
can **execute** in the real world!

Local models

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(f(\mathbf{x}_t, \mathbf{u}_t), \Sigma)$$

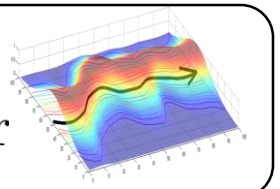
$$f(\mathbf{x}_t, \mathbf{u}_t) \approx \mathbf{A}_t \mathbf{x}_t + \mathbf{B}_t \mathbf{u}_t$$

$$\mathbf{A}_t = \frac{df}{d\mathbf{x}_t} \qquad \mathbf{B}_t = \frac{df}{d\mathbf{u}_t}$$



What controller to execute?

improve controller



iLQR produces: $\hat{\mathbf{x}}_t,\ \hat{\mathbf{u}}_t,\ \mathbf{K}_t,\ \mathbf{k}_t$

$$\mathbf{u}_t = \mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t$$

Version 0.5: $p(\mathbf{u}_t|\mathbf{x}_t) = \delta(\mathbf{u}_t = \hat{\mathbf{u}}_t)$

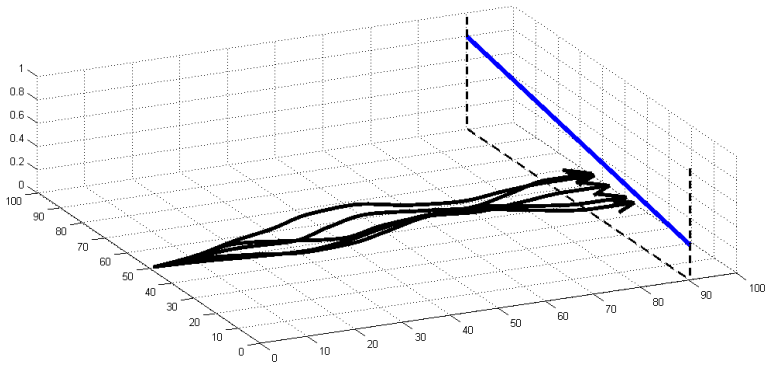
Doesn't correct deviations or drift

Version 1.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \delta(\mathbf{u}_t = \mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t)$

Better, but maybe a little too good?

Version 2.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$

Add noise so that all samples don't look the same!



What controller to execute?

Version 2.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$

Set $\Sigma_t = \mathbf{Q}^{-1}_{\mathbf{u}_t,\mathbf{u}_t}$

$Q(\mathbf{x}_t, \mathbf{u}_t)$ is the cost to go: total cost we get after taking an action

$$Q(\mathbf{x}_t, \mathbf{u}_t) = \text{const} + \frac{1}{2}\begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix}^T \mathbf{Q}_t \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix} + \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix}^T \mathbf{q}_t$$

$\mathbf{Q}_{\mathbf{u}_t,\mathbf{u}_t}$ is big if changing $\mathbf{u}_t$ changes the Q-value a lot!

If $\mathbf{u}_t$ changes Q-value a lot, don't vary $\mathbf{u}_t$ so much

Only act randomly when it minimally affects the cost to go

What controller to execute?

Version 2.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$

Set $\Sigma_t = \mathbf{Q}^{-1}_{\mathbf{u}_t,\mathbf{u}_t}$

Standard LQR solves $\min \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t)$

Linear-Gaussian solution solves $\min \sum_{t=1}^{T} E_{(\mathbf{x}_t,\mathbf{u}_t)\sim p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t, \mathbf{u}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))]$

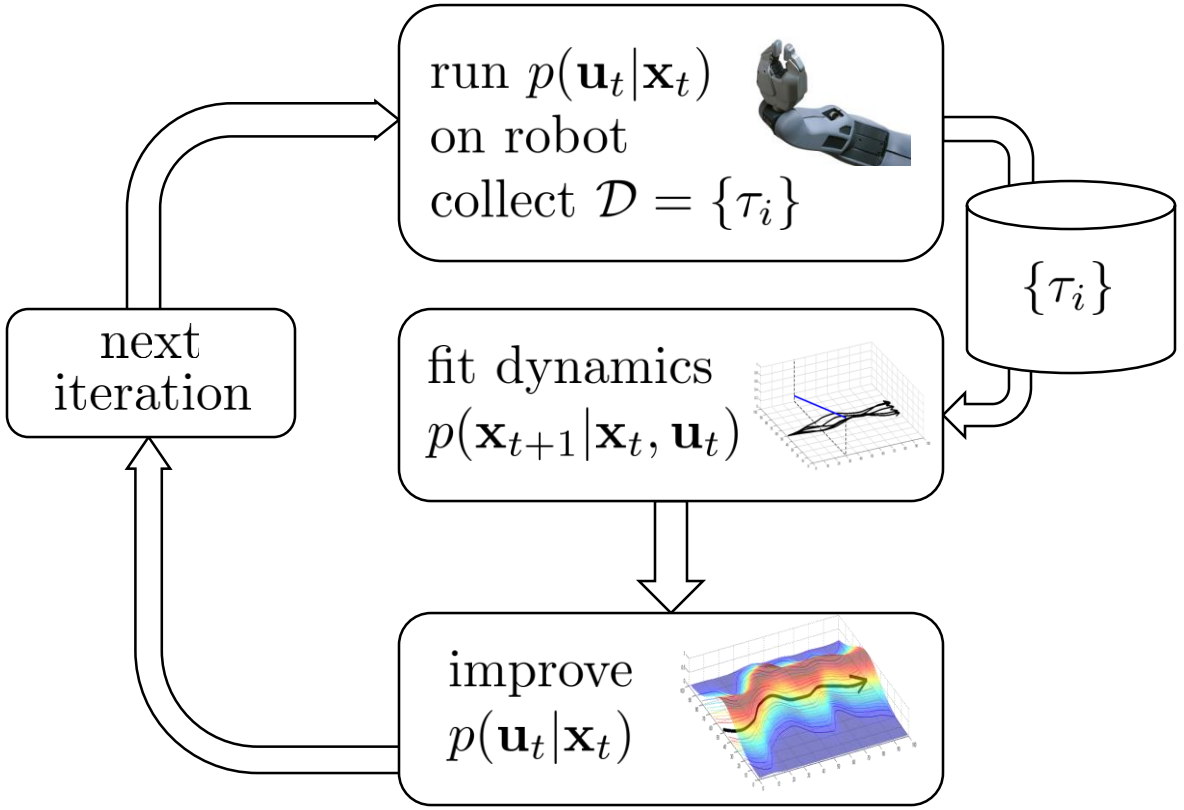
This is the *maximum entropy* solution: act as randomly as possible while minimizing cost

Local models

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(f(\mathbf{x}_t, \mathbf{u}_t), \Sigma)$$

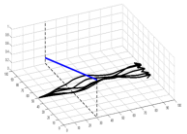
$$f(\mathbf{x}_t, \mathbf{u}_t) \approx \mathbf{A}_t \mathbf{x}_t + \mathbf{B}_t \mathbf{u}_t$$

$$\mathbf{A}_t = \frac{df}{d\mathbf{x}_t} \qquad \mathbf{B}_t = \frac{df}{d\mathbf{u}_t}$$



How to fit the dynamics?

fit dynamics
$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$



$\{(\mathbf{x}_t, \mathbf{u}_t, \mathbf{x}_{t+1})_i\}$

Version 1.0: fit $p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$ at each time step using linear regression

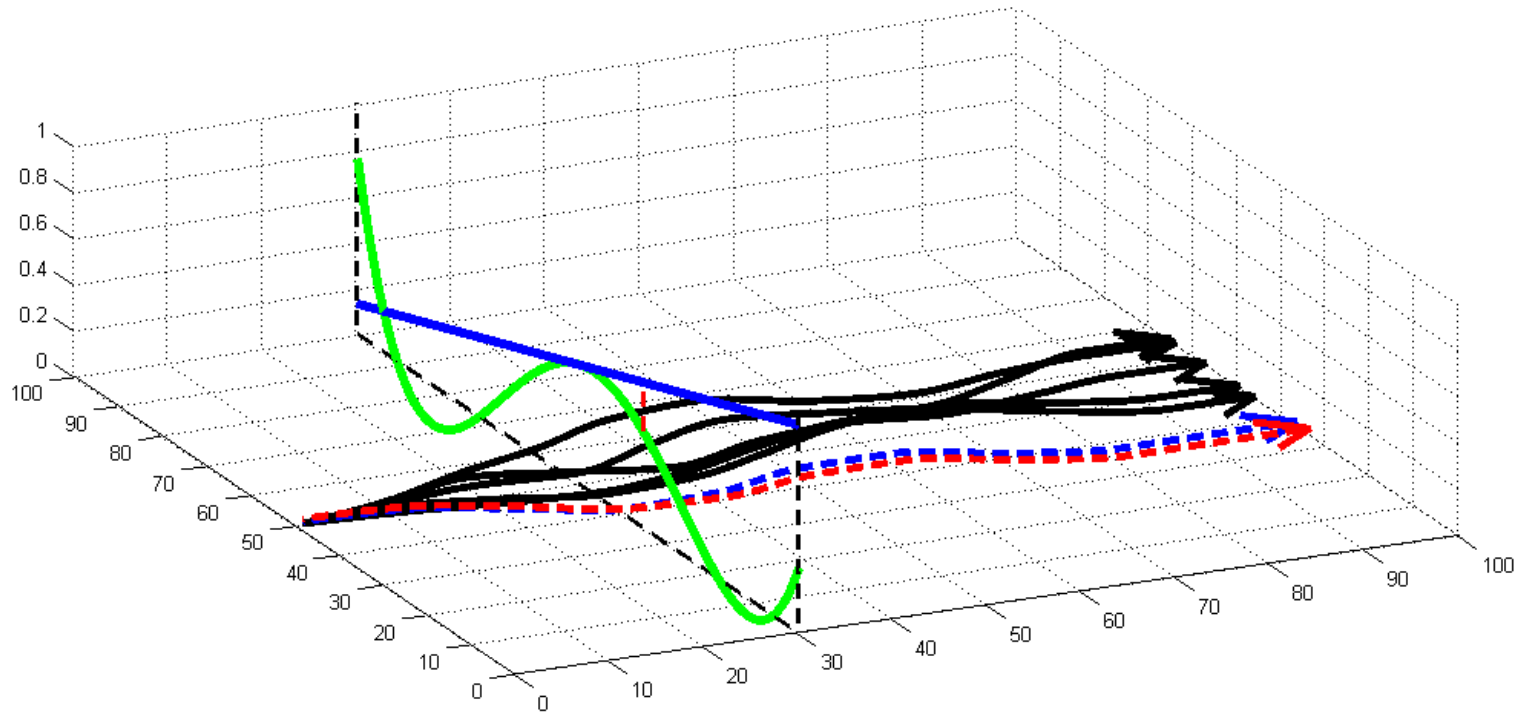
$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(\mathbf{A}_t\mathbf{x}_t + \mathbf{B}_t\mathbf{u}_t + \mathbf{c}, \mathbf{N}_t) \qquad \mathbf{A}_t \approx \frac{df}{d\mathbf{x}_t} \qquad \mathbf{B}_t \approx \frac{df}{d\mathbf{u}_t}$$

Can we do better?

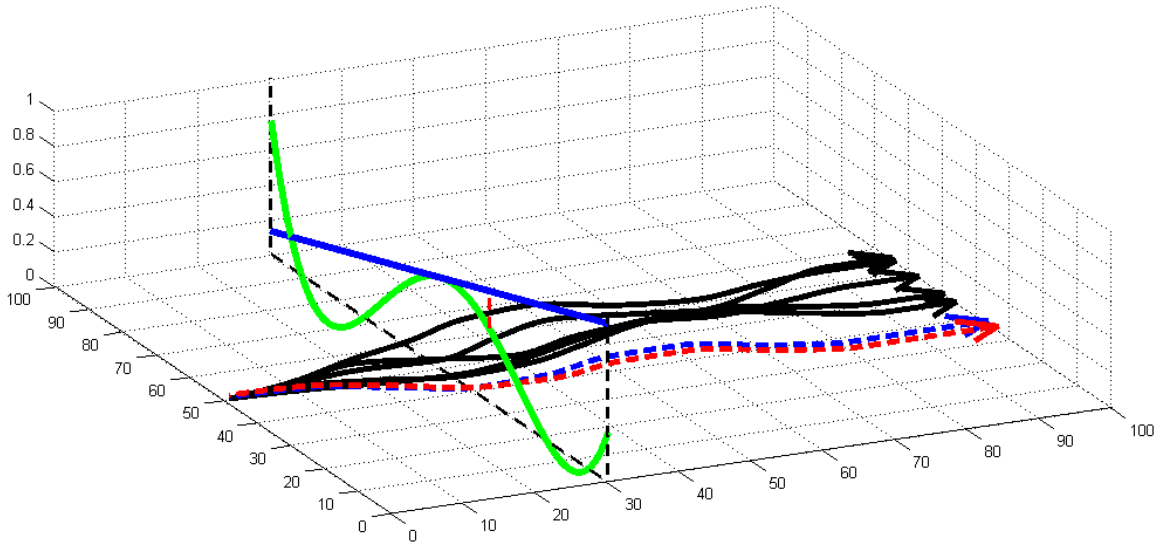
Version 2.0: fit $p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$ using *Bayesian* linear regression

Use your favorite *global* model as prior (GP, deep net, GMM)

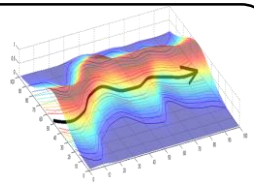
What if we go too far?



How to stay close to old controller?



improve $p(\mathbf{u}_t|\mathbf{x}_t)$



$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$$

$$p(\tau) = p(\mathbf{x}_1)\prod_{t=1}^{T} p(\mathbf{u}_t|\mathbf{x}_t)p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$$

What if the new $p(\tau)$ is "close" to the old one $\bar{p}(\tau)$?

If trajectory distribution is close, then dynamics will be close too!

What does "close" mean? $D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) \leq \epsilon$

KL-divergences between trajectories

- Not just for trajectory optimization – really important for model-free policy search too! More on this in later lectures

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = E_{p(\tau)}[\log p(\tau) - \log \bar{p}(\tau)]$$

$$p(\tau) = p(\mathbf{x}_1)\prod_{t=1}^{T} p(\mathbf{u}_t|\mathbf{x}_t)p(\mathbf{x}_{t+1}|\mathbf{x}_t,\mathbf{u}_t) \qquad \bar{p}(\tau) = p(\mathbf{x}_1)\prod_{t=1}^{T} \bar{p}(\mathbf{u}_t|\mathbf{x}_t)p(\mathbf{x}_{t+1}|\mathbf{x}_t,\mathbf{u}_t)$$

dynamics & initial state are the same!

$$\log p(\tau) - \log \bar{p}(\tau) = \log p(\mathbf{x}_1) + \sum_{t=1}^{T} \log p(\mathbf{u}_t|\mathbf{x}_t) + \log p(\mathbf{x}_{t+1}|\mathbf{x}_t,\mathbf{u}_t)$$

$$- \log p(\mathbf{x}_1) + \sum_{t=1}^{T} - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \log p(\mathbf{x}_{t+1}|\mathbf{x}_t,\mathbf{u}_t)$$

KL-divergences between trajectories

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = E_{p(\tau)}[\log p(\tau) - \log \bar{p}(\tau)]$$

$$\log p(\tau) - \log \bar{p}(\tau) = \cancel{\log p(\mathbf{x}_1)} + \sum_{t=1}^{T} \log p(\mathbf{u}_t|\mathbf{x}_t) + \cancel{\log p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)}$$

$$\cancel{- \log p(\mathbf{x}_1)} + \sum_{t=1}^{T} - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) \cancel{- \log p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)}$$

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = E_{p(\tau)}\left[\sum_{t=1}^{T} \log p(\mathbf{u}_t|\mathbf{x}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)\right]$$

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t, \mathbf{u}_t)}\left[\log p(\mathbf{u}_t|\mathbf{x}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)\right]$$

KL-divergences between trajectories

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}\left[\log p(\mathbf{u}_t|\mathbf{x}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)\right]$$

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}\left[-\log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)\right] + E_{p(\mathbf{x}_t)}[\underbrace{E_{p(\mathbf{u}_t|\mathbf{x}_t)}[\log p(\mathbf{u}_t|\mathbf{x}_t)]}]$$

negative entropy

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}\left[-\log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))\right]$$

KL-divergences between trajectories

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[-\log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))]$$

Reminder: Linear-Gaussian solves $\min \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t,\mathbf{u}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))]$

$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$$

If we can get D_{KL} into the cost, we can just use iLQR!

But how?

We want a constraint: $D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) \leq \epsilon$

Digression: dual gradient descent

$$\min_{\mathbf{x}} f(\mathbf{x}) \text{ s.t. } C(\mathbf{x}) = 0$$

$$\mathcal{L}(\mathbf{x}, \lambda) = f(\mathbf{x}) + \lambda C(\mathbf{x})$$

$$g(\lambda) = \inf_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda)$$

$$\lambda \leftarrow \arg\max_{\lambda} g(\lambda)$$

how to maximize? Compute the gradient!

Digression: dual gradient descent

$$\min_{\mathbf{x}} f(\mathbf{x}) \text{ s.t. } C(\mathbf{x}) = 0 \qquad \mathcal{L}(\mathbf{x}, \lambda) = f(\mathbf{x}) + \lambda C(\mathbf{x})$$

$$g(\lambda) = \inf_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda)$$

$$g(\lambda) = \mathcal{L}(\mathbf{x}^\star(\lambda), \lambda)$$

$$\frac{dg}{d\lambda} = \cancel{\frac{d\mathcal{L}}{d\mathbf{x}^\star}\frac{d\mathbf{x}^\star}{d\lambda}} + \frac{d\mathcal{L}}{d\lambda} \qquad \text{if } \mathbf{x}^\star = \arg\min_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda), \text{ then } \frac{d\mathcal{L}}{d\mathbf{x}^\star} = 0!$$

Digression: dual gradient descent

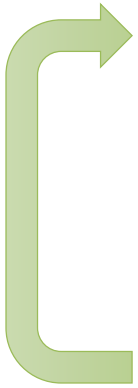
$$\min_{\mathbf{x}} f(\mathbf{x}) \text{ s.t. } C(\mathbf{x}) = 0$$

$$\mathcal{L}(\mathbf{x}, \lambda) = f(\mathbf{x}) + \lambda C(\mathbf{x})$$

$$g(\lambda) = \mathcal{L}(\mathbf{x}^\star(\lambda), \lambda)$$

$$\mathbf{x}^\star = \arg\min_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda)$$

$$\frac{dg}{d\lambda} = \frac{d\mathcal{L}}{d\lambda}(\mathbf{x}^\star, \lambda)$$



1. Find $\mathbf{x}^\star \leftarrow \arg\min_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda)$
2. Compute $\frac{dg}{d\lambda} = \frac{d\mathcal{L}}{d\lambda}(\mathbf{x}^\star, \lambda)$
3. $\lambda \leftarrow \lambda + \alpha \frac{dg}{d\lambda}$

DGD with iterative LQR

This is the constrained problem we want to solve:

$$\min_p \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t,\mathbf{u}_t)] \text{ s.t. } D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) \leq \epsilon$$

$$D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}\left[-\log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))\right]$$

$$\mathcal{L}(p,\lambda) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t,\mathbf{u}_t) - \lambda \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \lambda\mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))] - \lambda\epsilon$$

DGD with iterative LQR

$$\min_p \sum_{t=1}^{T} E_{p(\mathbf{x}_t, \mathbf{u}_t)}[c(\mathbf{x}_t, \mathbf{u}_t)] \text{ s.t. } D_{\mathrm{KL}}(p(\tau) \| \bar{p}(\tau)) \leq \epsilon$$

$$\mathcal{L}(p, \lambda) = \sum_{t=1}^{T} E_{p(\mathbf{x}_t, \mathbf{u}_t)}[c(\mathbf{x}_t, \mathbf{u}_t) - \lambda \log \bar{p}(\mathbf{u}_t | \mathbf{x}_t) - \lambda \mathcal{H}(p(\mathbf{u}_t | \mathbf{x}_t))] - \lambda \epsilon$$

1. Find $p^\star \leftarrow \arg\min_p \mathcal{L}(p, \lambda)$
2. Compute $\frac{dg}{d\lambda} = \frac{d\mathcal{L}}{d\lambda}(p^\star, \lambda)$
3. $\lambda \leftarrow \lambda + \alpha \frac{dg}{d\lambda}$

this is the hard part,
everything else is easy!

DGD with iterative LQR

1. Find $p^\star \leftarrow \arg\min_p \mathcal{L}(p, \lambda)$

$$\min_p \sum_{t=1}^{T} E_{p(\mathbf{x}_t, \mathbf{u}_t)}[c(\mathbf{x}_t, \mathbf{u}_t) - \lambda \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \lambda \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))] - \lambda\epsilon$$

Reminder: Linear-Gaussian solves $\min \sum_{t=1}^{T} E_{p(\mathbf{x}_t, \mathbf{u}_t)}[c(\mathbf{x}_t, \mathbf{u}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))]$

$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$$

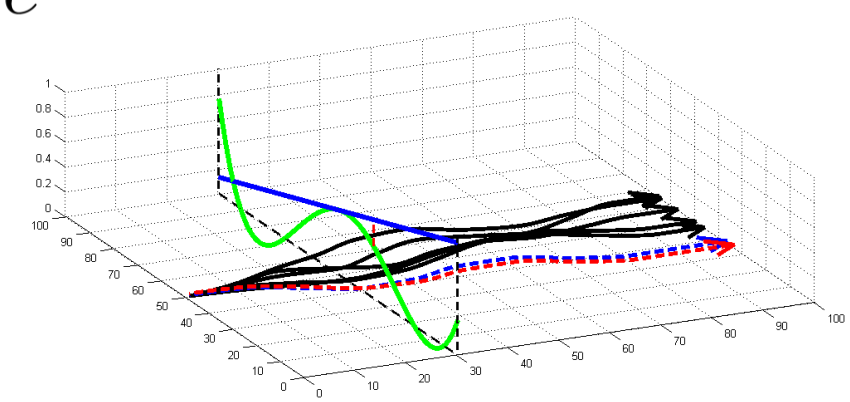
$$\min_p \sum_{t=1}^{T} E_{p(\mathbf{x}_t, \mathbf{u}_t)} \left[\frac{1}{\lambda} c(\mathbf{x}_t, \mathbf{u}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t)) \right]$$

Just use LQR with cost $\tilde{c}(\mathbf{x}_t, \mathbf{u}_t) = \frac{1}{\lambda} c(\mathbf{x}_t, \mathbf{u}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)$

DGD with iterative LQR

$$\min_p \sum_{t=1}^{T} E_{p(\mathbf{x}_t,\mathbf{u}_t)}[c(\mathbf{x}_t,\mathbf{u}_t)] \text{ s.t. } D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) \leq \epsilon$$

1. Set $\tilde{c}(\mathbf{x}_t,\mathbf{u}_t) = \frac{1}{\lambda} c(\mathbf{x}_t,\mathbf{u}_t) - \log \bar{p}(\mathbf{u}_t|\mathbf{x}_t)$
2. Use LQR to find $p^\star(\mathbf{u}_t|\mathbf{x}_t)$ using $\tilde{c}$
3. $\lambda \leftarrow \lambda + \alpha(D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) - \epsilon)$



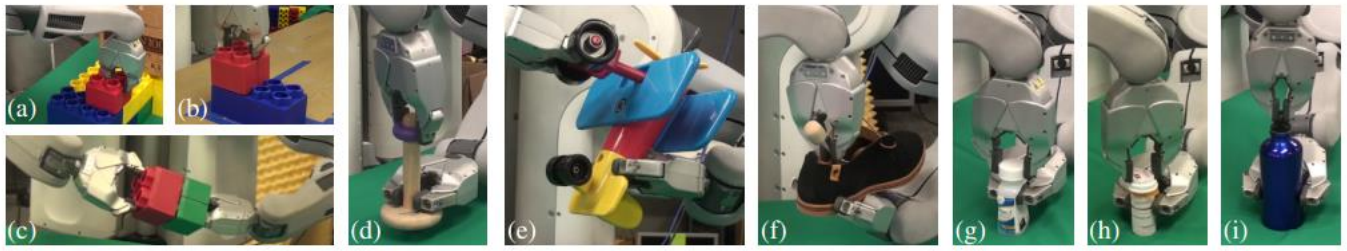
Trust regions & trajectory distributions

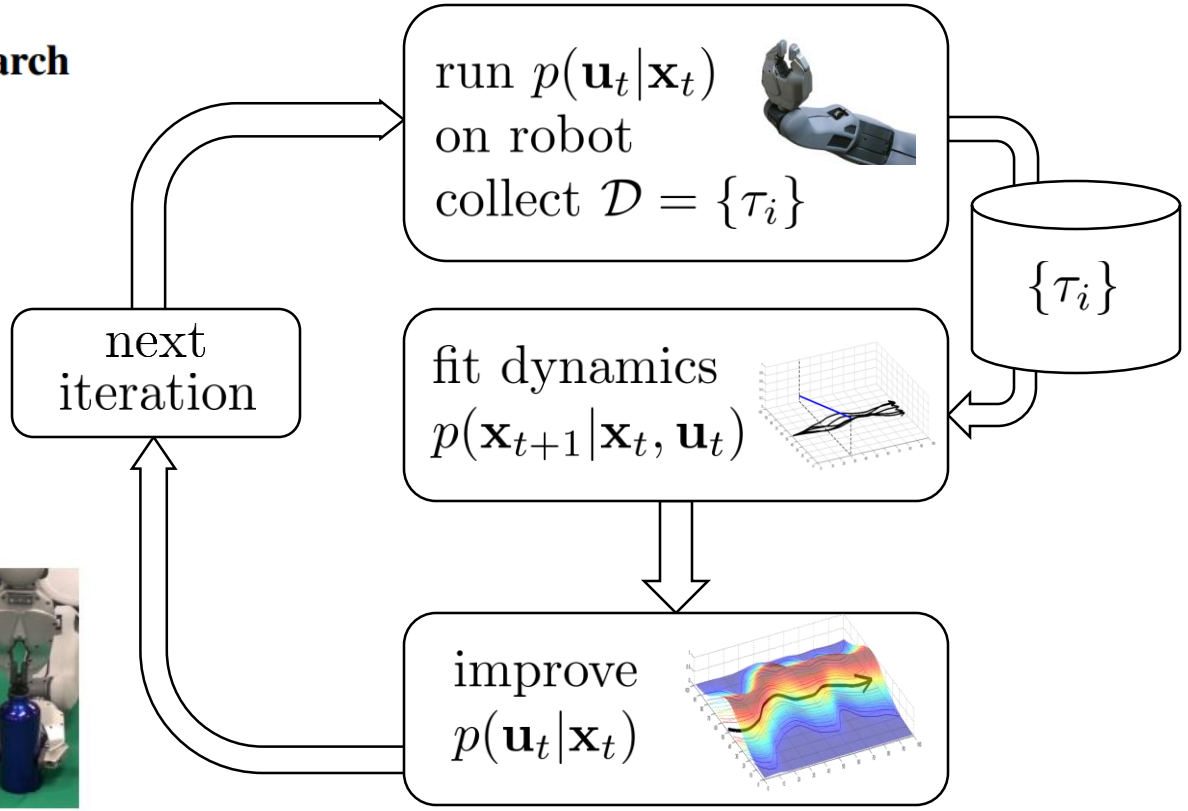
- Bounding KL-divergences between two policies or controllers, whether linear-Gaussian or more complex (e.g. neural networks) is really useful
- Bounding KL-divergence between policies is equivalent to bounding KL-divergences between trajectory distributions
- We'll use this later in the course in model-free RL too!

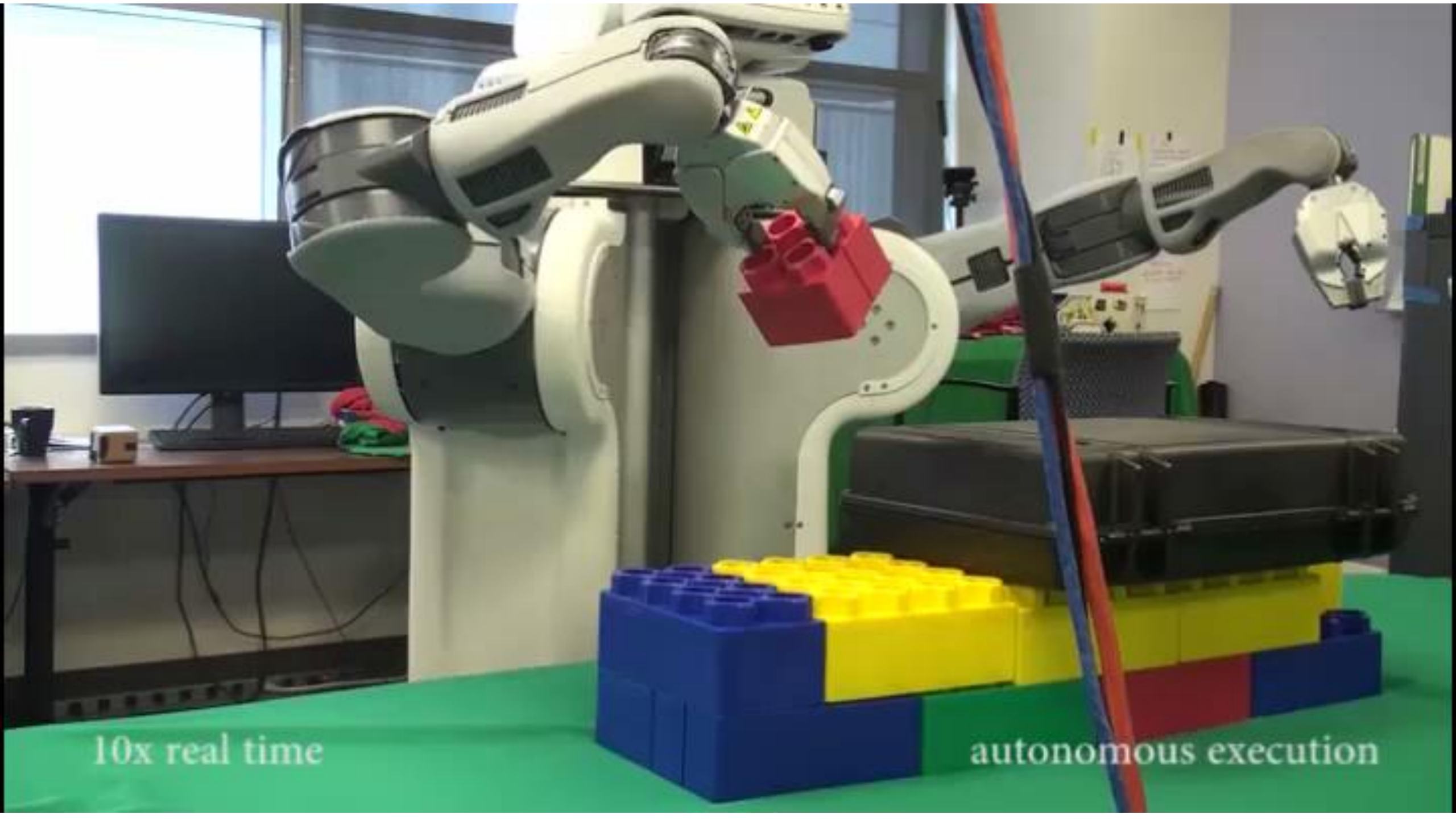
Case study: local models & iterative LQR

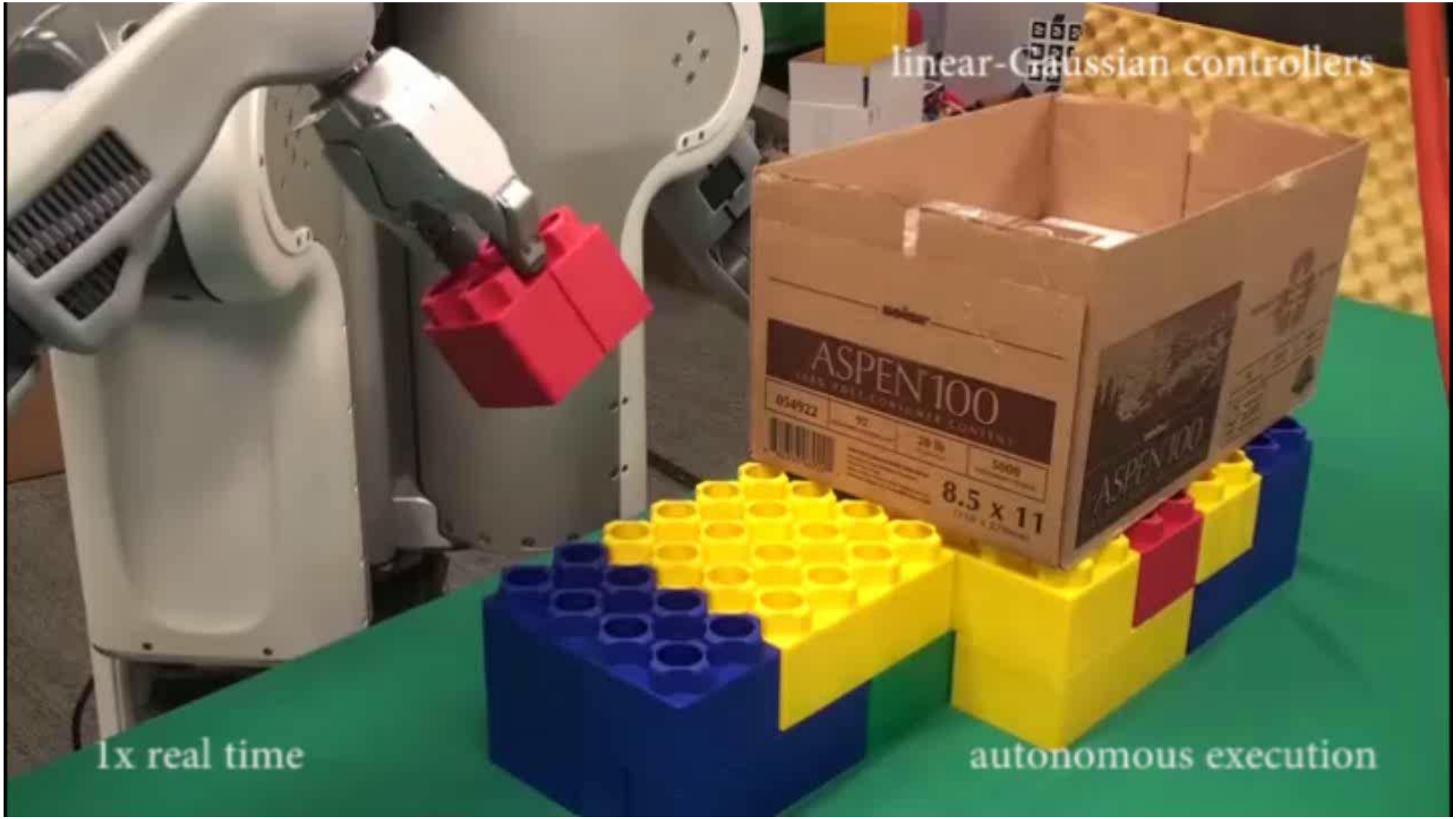
Learning Contact-Rich Manipulation Skills with Guided Policy Search

Sergey Levine, Nolan Wagener, Pieter Abbeel

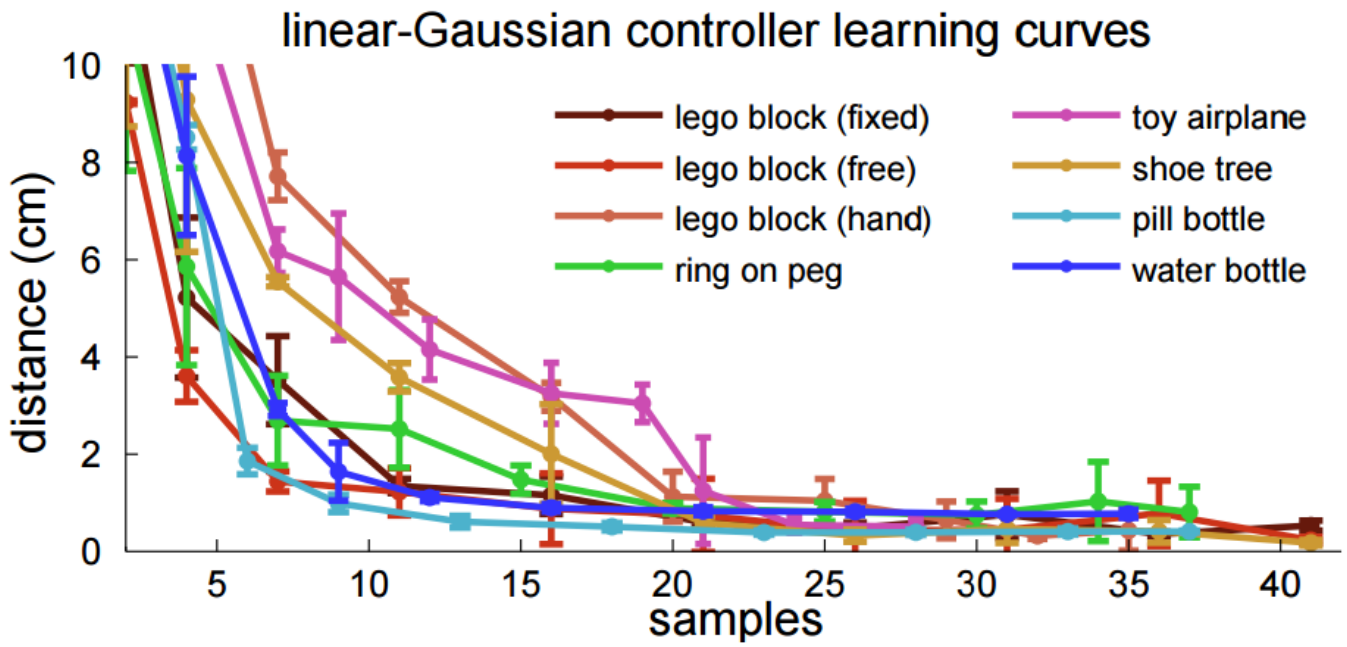


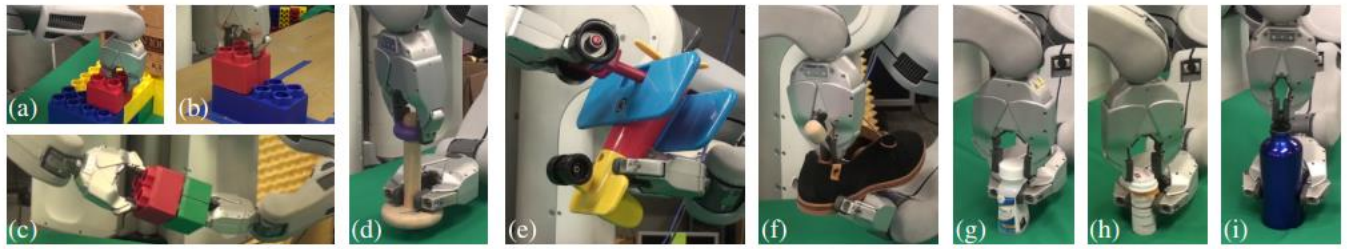



10x real time
autonomous execution


linear-Gaussian controllers
ASPEN 100
8.5 x 11
1x real time
autonomous execution

Case study: local models & iterative LQR

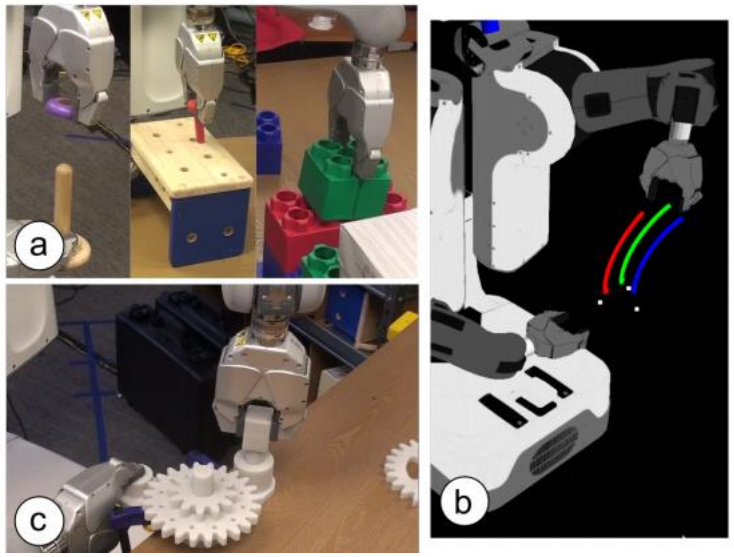


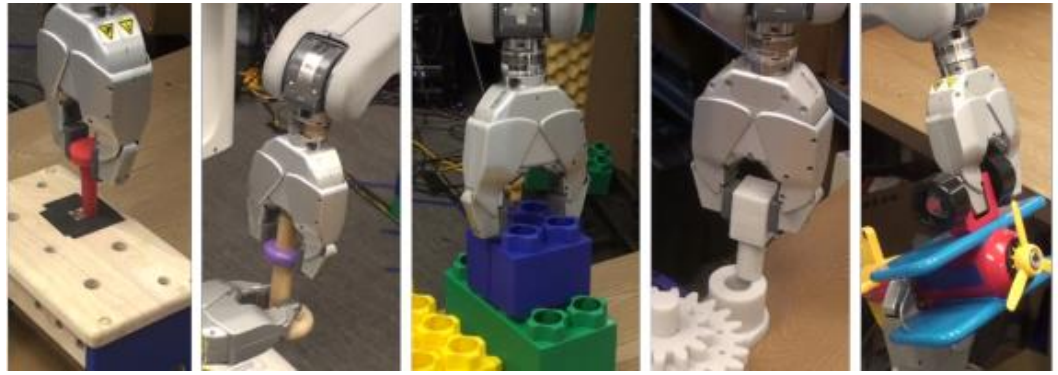


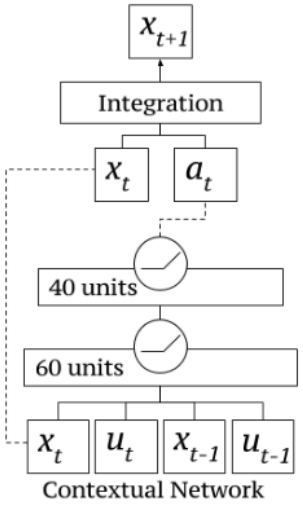
Case study: combining global and local models

One-Shot Learning of Manipulation Skills with Online Dynamics Adaptation and Neural Network Priors

Justin Fu, Sergey Levine, Pieter Abbeel





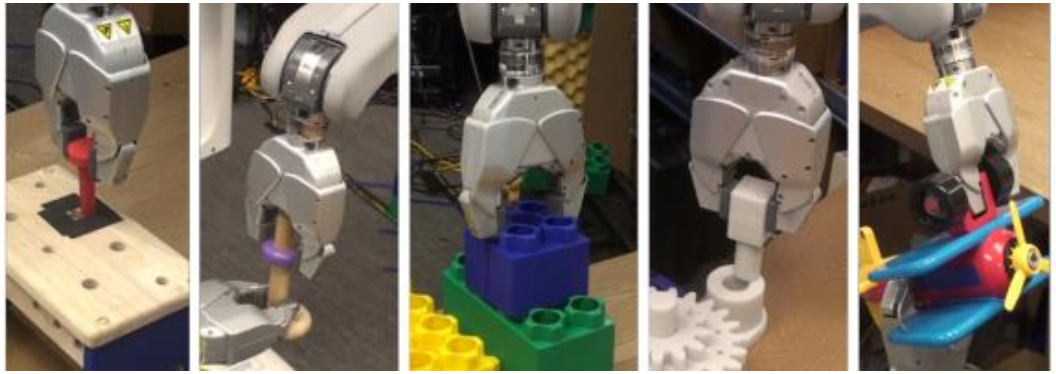


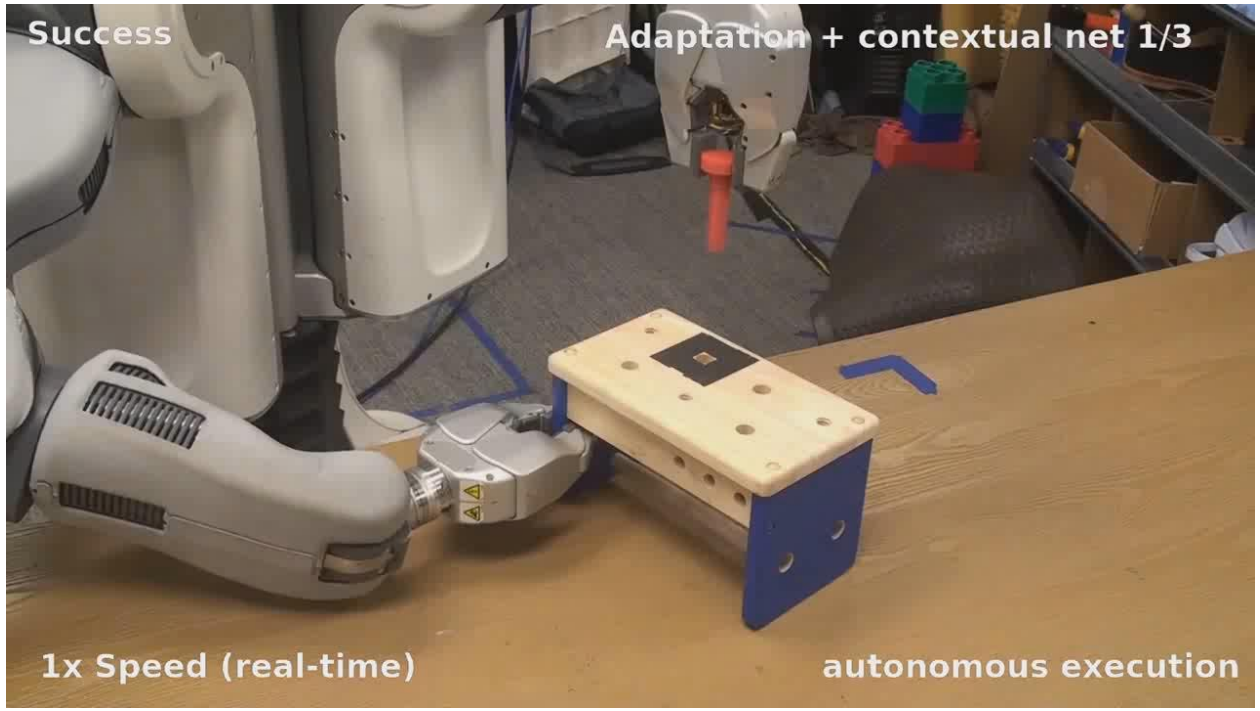
prior:
$\mathbf{\Phi}$, μ_0

empirical
estimate:
$\hat{\Sigma}$, $\hat{\mu}$

recent
experience
$(\mathbf{x}_t, \mathbf{u}_t, \mathbf{x}_{t+1})$

posterior:
Σ, μ




Success
Adaptation + contextual net 1/3
1x Speed (real-time)
autonomous execution