Gaussian Belief Space Planning for Imprecise Articulated Robots
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Motivation: Facilitate reliable operation of cost-effective robots that use:
- Imprecise actuation mechanisms such as serial elastic actuators and cables
- Inaccurate encoders and sensors such as gyro and accelerometers to sense robot state

Problem: Reliable manipulation and navigation requires such robots to explicitly perform information gathering actions to minimize effects of uncertainty

Formally, this can be modeled as a Partially Observable Markov Decision Process (POMDP); computing globally optimal solutions (policies) is computationally intractable

Approach: Compute locally optimal trajectories in belief space (space of probability distributions over states) [2]

Contributions:
- Prior work approximates robot geometry as points or spheres; we consider articulated robots
- Sigma Hulls for probabilistic collision avoidance
- Model predictive control (MPC) during execution in Gaussian belief space using efficient SQP-based trajectory optimization methods [3]

Trajectory optimization in Gaussian belief space:

Gaussian belief state in joint space: \( \mathbf{b}_t = \begin{bmatrix} \mu_t \\ \sqrt{\Sigma_t} \end{bmatrix} \)

Optimization problem:

Variables (beliefs, control inputs): \( \mathbf{b}_1, \ldots, \mathbf{b}_T, \mathbf{u}_1, \ldots, \mathbf{u}_{T-1} \)

Minimize: \( \| \mu_T - \mu_{\text{target}} \| + \sum_{t=1}^{T} \text{tr}[\Sigma_t] \)

Subject to constraints:
- Belief dynamics (Unscented Kalman Filter [1]) are satisfied: \( \mathbf{b}_{t+1} = \text{UKF}(\mathbf{b}_t, \mathbf{u}_t) \)
- Control inputs are feasible: \( \mathbf{u}_t \in \mathcal{F}_t \)
- Trajectory is \( \sigma \)-standard deviations safe (all pairs of links and obstacles): \( \text{sign dist}(\mathcal{L}_i, \mathcal{O}_j, \sigma) \geq 0 \quad \forall t \in \{1, \ldots, T\} \)

Future Work:
- Planning in uncertain environments
- Non-Gaussian belief spaces
- Physical experiments with the Raven surgical robot

\[ \begin{align*}
\text{Scenario: Imprecise 7-DOF robot operating in a constrained environment. The robot localizes itself based on distance measurements from a wall using a sensor mounted on the end-effector.}

\text{Uncertainty unaware RRT plan}

\text{Belief space plan (35 dimensional space, ~3s)}

\text{Re-plan at every time step during execution (MPC)}

\text{Open-loop execution (probability of collision: 83%)}

\text{Closed-loop execution (probability of collision: 6%)}
\end{align*} \]